# Surface Integrals of Vector Fields- HW Problems

Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ using an upward/outward pointing normal to the surface.

1. $\vec{F}(x, y, z) = z\vec{\imath} - 2\vec{\jmath} + y\vec{k}$, $S$ is the plane $x + y + z = 1$, where $x \geq 0,\ y \geq 0,\ z \geq 0.$

2. $\vec{F}(x, y, z) = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, $S$ is given by $z = 4 - x^2 - y^2$, where $z \geq 0.$

3. $\vec{F}(x, y, z) = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, where $S$ is the upper unit hemisphere.

4. $\vec{F}(x, y, z) = -3\vec{\imath} + 4\vec{\jmath} + 5\vec{k}$, $S$ is given by $z = x^2 + y^2$, where $x^2 + y^2 \leq 9.$

5. $\vec{F}(x, y, z) = x\vec{\imath} + y\vec{\jmath} - z^2\vec{k}$, where $S$ is given by $\vec{\Phi}(u, v) = < 2\cos(u), 2\sin(u), v >$; with $0 \leq u \leq 2\pi,\ \ 0 \leq v \leq 2$

6. $\vec{F}(x, y, z) = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, where $S$ is the closed surface made up of the upper unit hemisphere $x^2 + y^2 + z^2 = 1,\ z \geq 0$ and the unit disk in the $x$-$y$ plane $x^2 + y^2 \leq 1.$

7. Find the flux of the vector field

$$\vec{F}(x, y, z) = (xy^2)\vec{\imath} + (x^2y)\vec{\jmath} + (\tfrac{1}{3}z^3)\vec{k}$$

out of the unit sphere.

8. Find the flux of the vector field $\vec{F}(x,y,z) = (-\frac{\sqrt{2}}{2})\vec{\imath} + (\frac{\sqrt{2}}{2})\vec{k}$ through the portion of the cone given by $z = \sqrt{x^2+y^2}$ ; where $x^2 + y^2 \leq 1$.

9. Find the flux across the surface $S$ of $\vec{F}(x,y,z) =< -\frac{3}{2}x\,, -\frac{3}{2}y,\ z >$ where $S$ is given by $z = 2 + x^2 + y^2$; where $3 \leq z \leq 6$.

10. Evaluate $\iint_S \vec{F} \cdot \vec{n} dA$, where $\vec{F}(x,y,z) = \vec{\imath} + \vec{\jmath} + 2z(x^2+y^2)^2\vec{k}$, and $S$ is the boundary of the solid cylinder given by $x^2 + y^2 \leq 1$, $0 \leq z \leq 2$.