

Surface Integrals of Scalar Functions- HW Problems

Evaluate the following surface integrals.

1. $\iint_S (z + 3)dS$; where S is given by $\vec{\Phi}(u, v) = \langle u, \frac{v}{2}, \frac{v}{4} \rangle$; $(u, v) \in [0,3] \times [0,4]$.
2. $\iint_S (x^2)dS$; where S is portion of the cylinder in \mathbb{R}^3 given by $x^2 + y^2 = 9$, $1 \leq z \leq 3$.
3. $\iint_S (xz - yz)dS$; where S is portion of the plane in \mathbb{R}^3 $z = x + y + 2$, that lies inside the cylinder $x^2 + y^2 = 1$.
4. $\iint_S (z)dS$; where S is the interior of the triangle with vertices at $(2,0,0)$, $(0,1,0)$, and $(1,0,1)$.
5. $\iint_S \left(\frac{2xy}{z}\right) dS$; where S is given in \mathbb{R}^3 by $z = x^2 + y^2$ where $4 \leq x^2 + y^2 \leq 9$, $x \geq 0$, $y \geq 0$.
6. $\iint_S (\sqrt{x^2 + y^2 + z^2})dS$; where S is given in \mathbb{R}^3 by $z = \sqrt{x^2 + y^2}$ where $x^2 + y^2 \leq 1$.
7. $\iint_S (z + 1)dS$; where S is given in \mathbb{R}^3 by $z = 4 - x^2 - y^2$ where $x^2 + y^2 \leq 4$.

8. $\iint_S (z^2) dS$; where S is the upper unit hemisphere.

9. A helicoid, S , is parametrized by

$$\vec{\Phi}(r, \theta) = \langle r \cos(\theta), r \sin(\theta), \theta \rangle; \text{ where } 0 \leq r \leq 4, 0 \leq \theta \leq 2\pi.$$

Suppose the density at $(x, y, z) \in S$ is given by

$$\rho(x, y, z) = (x^2 + y^2)^{\frac{3}{2}} = (r^2)^{\frac{3}{2}}. \text{ Find the mass of the helicoid.}$$