

Integrating Differential Forms over Subsets of \mathbb{R}^3 -HW Problems

Evaluate.

1. $\int_C \omega$; $\omega = xyzdx + yzdy + zdz,$

$$\vec{c}(t) = \langle t, t^2, t^3 \rangle, \quad 0 \leq t \leq 1$$

2. $\int_C f\omega$; $\omega = (x^2 + y^2 + z^2)dx + xydy + yzdz,$

$$f(x, y, z) = y$$

$$\vec{c}(t) = \langle 1, t, t^3 \rangle, \quad 0 \leq t \leq 1$$

3. $\iint_S \eta$; $\eta = xydxdy$

S is the portion of the sphere $x^2 + y^2 + z^2 = 1,$

and $x \geq 0, y \geq 0, z \geq 0.$

4. $\iint_S \eta$; $\eta = y^2dxdy + xdydz$

S is the portion of the cone $\vec{\Phi}(r, \theta) = \langle r\cos(\theta), r\sin(\theta), r \rangle;$

$0 \leq r \leq 2, \quad 0 \leq \theta \leq 2\pi.$

5. $\iiint_W \varphi$; $\varphi = (x^2 + y^2 + z)dxdydz$

$$W = [0,1] \times [0,3] \times [1,2].$$