Conservative Vector Fields- HW Problems

In problems 1-4 determine which of the vector fields is conservative (ie a gradient vector field). If the vector field is conservative find a function f such that $\nabla f = \vec{F}$.

1. $\vec{F}(x,y) = \big(2xycos(y)\big)\vec{\imath} - (x^2\sin(y))\vec{\jmath}$

2. $\vec{F}(x,y) =< e^y + ye^x,\ e^x + xe^y >$

3. $\vec{F}(x,y,z) =< 2xy,\ \ x^2 + 2yz^3,\ 3y^2z^2 + 1 >$

4. $\vec{F}(x,y,z) = \big(sin(y)\big)\vec{\imath} - (xcos(y) + \cos(z))\vec{\jmath} - (ysin(z))\vec{k}$

5. Let $\vec{F}(x,y) = \big(4xcos(y)\big)\vec{\imath} - (2x^2\sin(y))\vec{\jmath}$.

a. Find a function f such that $\nabla f = \vec{F}$.

b. Evaluate the integral $\int_c \vec{F}\cdot d\vec{s}$ where $\vec{c}(t) =< \cos^7 t\ ,\ t^4 >$; $0 \le t \le \frac{\pi}{2}$.

6. Let $\vec{F}(x,y,z) = (y^2)\vec{\imath} + (2xy + e^{3z})\vec{\jmath} + (3ye^{3z})\vec{k}$

a. Find a function f such that $\nabla f = \vec{F}$.

b. Evaluate the integral $\int_c \vec{F}\cdot d\vec{s}$ where

$$\vec{c}(t) =< t^4,\ \sqrt[3]{t},\ \ln(t+2) >;\quad 0 \le t \le 1.$$

7. Let $\vec{F}(x,y,z) = (2xyz)\vec{\imath} + (x^2z)\vec{\jmath} + (x^2y)\vec{k}$. Evaluate $\int_c \vec{F} \cdot d\vec{s}$ where $\vec{c}(t) =< t^4 + 1, \cos(\pi t),\ te^{(1-t)} >; \quad 0 \leq t \leq 1.$

8. Let $\vec{F}(x,y) =< \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} >.$

a. Evaluate the integral $\int_c \vec{F} \cdot d\vec{s}$ where $\vec{c}(t) =< \cos(t),\ \sin(t) >;$ $0 \leq t \leq 2\pi.$

b. Show that if $P(x,y) = -\frac{y}{(x^2+y^2)}$ and $Q(x,y) = \frac{x}{x^2+y^2}$ then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$.

c. However, by “a”, $\vec{F}$ is not a conservative vector field. Does this violate the conservative vector field theorem for the plane? Explain.

9. Show that the following vector fields are conservative and calculate $\int_c \vec{F} \cdot d\vec{s}$ for the given curve c.

a. $\vec{F}(x,y) = (x^2 + y^2)\vec{\imath} + (2xy)\vec{\jmath}$; c is the triangle with vertices at $(-1,0), (5,0)$, and $(2,3)$, oriented counterclockwise.

b. $\vec{F}(x,y) =< -y\sin(x) + \sin(y),\ \cos(x) + x\cos(y) >;$ $\vec{c}(t) =< \frac{\pi}{2}te^{(1-t)},\ 2\pi - \frac{3\pi}{2}t^9 >, \quad 0 \leq t \leq 1.$

10. Show that $\vec{F}(x,y,z) =< x\cos(y), -\sin(y),\ \sin(x) >$ is the curl of a vector field $\vec{G}$, without finding $\vec{G}$.

11. Show that $\vec{F}(x, y, z) = < x^3,\ -2x^2y, -x^2z >$ is the curl of a vector field $\vec{G}$. Find $\vec{G}(x, y, z)$ such that $\vec{F} = \nabla \times \vec{G}$.