Stokes' Theorem- HW Problems

1. Verify Stokes' theorem for $\vec{F}(x,y,z) = (z^2)\vec{\imath} + (x)\vec{\jmath} + (y^2)\vec{k}$ and $S = \{(x,y,z)|\ x^2 + y^2 + z^2 = 4,\ z \geq \sqrt{3}\}$ oriented as the graph of $z = \sqrt{4 - x^2 - y^2}$.

2. Verify Stokes' for $\vec{F}(x,y,z) = (-2y)\vec{\imath} + (x)\vec{\jmath} - (3)\vec{k}$ and S is the portion of the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 2$.

In problems 3-5 evaluate $\int_c \vec{F} \cdot d\vec{s}$ using Stokes' theorem. In each case c is oriented counterclockwise when viewed from above.

3. $\vec{F}(x,y,z) = (y^2)\vec{\imath} + (z)\vec{\jmath} + (-x + e^{\cos(z)})\vec{k}$; c is the triangle with vertices at $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

4. $\vec{F}(x,y,z) = (z)\vec{\imath} + (x^2)\vec{\jmath} + (y - \sin(z))\vec{k}$; c is the boundary of the helicoid given by $\overrightarrow{\Phi}(r,\theta) = < rcos(\theta), rsin(\theta), \theta >$; $0 \leq r \leq 1$, $0 \leq \theta \leq \frac{\pi}{2}$.

5. $\vec{F}(x,y,z) = (xy)\vec{\imath} + (z)\vec{\jmath} + (y)\vec{k}$; c is the intersection of the plane $z = 5 - x$ and the cylinder $x^2 + y^2 = 4$.

In problems 6 and 7 use Stokes' theorem to evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$.

6. $\vec{F}(x, y, z) = (x^2 z^2)\vec{\imath} + (y^2 z^2)\vec{\jmath} + (xyz)\vec{k}$; S is the portion of the sphere $x^2 + y^2 + z^2 = 9$ that lies inside the cylinder $x^2 + y^2 = 4$.

7. $\vec{F}(x, y, z) = (\cos(xy))\vec{\imath} - (e^z)\vec{\jmath} + (yz)\vec{k}$; S is the sphere $x^2 + y^2 + z^2 = 4$.

8. Let $\vec{F}(x, y, z) = (\mathrm{yz})\vec{\imath} + (e^{xz})\vec{\jmath} + (xyz)\vec{k}$. Suppose S_1 is the portion of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ that lies inside of the cylinder $x^2 + y^2 = 2$ and S_2 is the disk $x^2 + y^2 \leq 2,\ z = \sqrt{2}$. Show that $\iint_{S_1} (\nabla \times \vec{F}) \cdot d\vec{S} = \iint_{S_2} (\nabla \times \vec{F}) \cdot d\vec{S}$.