Green's Theorem- HW Problems

Verify Green's theorem in problems 1-3 by calculating both sides of the equality.

1. $P(x,y) = x - y, \quad Q(x,y) = x + y, \quad D = \{(x,y) |\, x^2 + y^2 \le 4\}$

2. $P(x,y) = xy, \quad Q(x,y) = x^2y^3, \quad c$ is the triangle with vertices at $(0,0)$, $(1,0)$, and $(1,3)$.

3. $P(x,y) = 2x^3 - y^3, \quad Q(x,y) = x^3 + y^3,$

   $D = \{(x,y) |\, 1 \le x^2 + y^2 \le 9\}$

4. Use Green's theorem to find the work done to move a particle counterclockwise around $x^2 + y^2 = 1$ if the force on the particle is given by $\vec{F}(x,y) = (3x + 4y^2)\vec{\imath} + (12xy)\vec{\jmath}$.

5. Use Green's theorem to find the area of the annulus given by $1 \le x^2 + y^2 \le 9$.

6. Use Green's theorem to find the area in $\mathbb{R}^2$ bounded by $y = 4 - x^2$ and $y = x - 2$.

In problems 7-9, evaluate the integrals using Green's theorem.

7. $\int_c (y^2 + e^{\sqrt{x}})dx + (2x^2 - \sin(y^2))dy$ where c is the triangle in $\mathbb{R}^2$ with vertices at $(0,0)$, $(1,1)$, and $(1,2)$ oriented counterclockwise.

8. $\int_c (2y^2 + xsin(x))dx + \left(x^3 - e^{\sqrt{y}}\right)dy$ where c is the rectangle in $\mathbb{R}^2$ with vertices at $(1,0)$, $(2,0)$, $(2,2)$ and $(1,2)$ oriented counterclockwise.

9. $\int_c (xy^2)dx - (yx^2)dy$ where c is the circle $x^2 + y^2 = 4$ oriented counterclockwise.

10. Verify the divergence theorem where $\vec{F}(x,y) = (x)\vec{\imath} - (xy^2)\vec{\jmath}$ and D is the disk $x^2 + y^2 \leq 4$.