The Riemann Integral- HW Problems

1. Give an example of a function $f(x)$ defined on $[0,1]$ that is not Riemann integrable but $|f(x)|$ is Riemann integrable.

2. A partition Q is called a refinement of a partition P if Q contains all of the point of P (and possibly others). Let f be a bounded function on $[a, b]$. Show that under a refinement that $U(f, Q) \leq U(f, P)$ and $L(f, Q) \geq L(f, P)$.

3. Let f be a bounded function on $[a, b]$. Using the result in problem 2, show that for any two partitions P, Q (where one partition is not necessarily a refinement of the other) we have: $L(f, Q) \leq U(f, P)$.

4. Let f be increasing on $[0,1]$. Let P be the partition of $[0,1]$ where each subinterval is of length $\frac{1}{n}$. Show that

$$U(f, P) - L(f, P) \leq \frac{1}{n}[f(1) - f(0)].$$