The Simple Approximation Theorem- HW Problems

1. Let $f(x) = |x| \qquad -1 < x < 1, \ \ x \neq 0$

$= \frac{1}{2} \qquad x = 0 \ \ or \ \ x = 1$

$= x - 1 \qquad 1 < x < 2.$

Approximate $f(x)$ on $(-1,2)$ by simple functions φ and ψ, where $\varphi \leq f \leq \ \psi$ and $0 \leq \psi - \varphi \leq 0.3$ on $(-1,2)$.

2. Let E and F be any sets. Show that

a. $\chi_{E \cap F} = \chi_E \cdot \chi_F$

b. $\chi_{E \cup F} = \chi_E + \chi_F - \chi_E \cdot \chi_F$

c. $\chi_{E^c} = 1 - \chi_E$.

3. Show that if φ and ψ are simple functions, so are $\varphi + \psi$, $\varphi\psi$, $\max(\varphi, \psi)$, and $\min(\varphi, \psi)$.

4. Write any measurable function f as the difference of two nonnegative measurable functions. Now prove a general Simple Approximation Theorem for any measurable function (not just a nonnegative one).