# Lebesgue Measurable Functions- HW Problems

1. Prove if $f$ is a monotonic increasing function on an open interval $I = (a, b)$, then $f$ is measurable.

2. A function $f: [a, b] \to \mathbb{R}$ is a step function if there are finitely many points $a = t_0 < t_1 < \cdots < t_n = b$ such that $f$ is constant on each of the open intervals $(t_i, t_{i+1})$ ($f$ can take any value at the points $t_i$). Prove any step function is measurable.

3. Prove the characteristic function $\chi_E$ is measurable if and only if $E$ is measurable, where $\chi_E: \mathbb{R} \to \mathbb{R}$ and $\chi_E(x) = 1$ if $x \in E$ and $\chi_E(x) = 0$ if $x \notin E$.

4. Let $E$ be a nonmeasurable subset of (0,1). Define $f$ on $(0,1)$ by $f(x) = x$ if $x \in E$ and $f(x) = -x$ if $x \notin E$. Prove that $f$ is not measurable yet $\{x \in (0,1) | f(x) = c\}$ is measurable for all $c \in \mathbb{R}$.