The Lebesgue Integral $\int_E f$: f Bounded, $m(E) < \infty$ - HW Problems

1. Find a sequence $\{f_n\}$ of nonnegative measurable functions on sets E_n, where $m(E_n) < \infty$, such that f_n converges to $f(x) = 0$ uniformly on $\bigcup_{n=1}^{\infty} E_n$, and $\lim_{n\to\infty} \int_{E_n} f_n = 1$. (Make sure you prove $f_n \to f$ uniformly on $\bigcup_{n=1}^{\infty} E_n$).

2. Suppose $m(E) = 0$. Show that if f is bounded on E then f is measurable and $\int_E f = 0$.

3. Let $m(E) < \infty$. Suppose that f is bounded and measurable on E. Show that for any measurable subset $B \subseteq E$ that $\int_B f = \int_E f \cdot \chi_B$.

4. Let $m(E) < \infty$. Suppose that f is bounded and measurable on E. Suppose that g is bounded and $g = f$ a.e. on E. Prove that $\int_E f = \int_E g$.