Composition of Linear Transformations- HW Problems

For problems 1-5 assume $A = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 1 \\ 3 & -2 & -1 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 1 \\ -2 & 3 \\ 3 & -1 \end{bmatrix}$. Find

1. $(A)(A)$
2. $(A)(B)$
3. $(C)(A)$
4. $(B^t)(A)$
5. $(B)(C)$

6. Suppose $T, U: \mathbb{R}^2 \to \mathbb{R}^2$ by

$$T(x_1, x_2) = (x_1 + 2x_2,\ 2x_1 - x_2)$$

$$U(x_1, x_2) = (x_1 + x_2,\ x_1 - x_2).$$

a. Find $T \circ U(x_1, x_2)$ and $U \circ T(x_1, x_2)$ (without using matrices).

b. Assuming the standard basis for $\mathbb{R}^2$ find matrix representations of $T,\ U,\ U \circ T,$ and $T \circ U$.

c. Show through matrix multiplication that the matrix representation of $U \circ T$ equals the product of the matrix representations of U and T and that the matrix representation of $T \circ U$ is the product of the matrix representations of T and U.

7. Suppose $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ by $T\big(p(x)\big) = x^2p''(x) + xp'(x)$ and $U: P_2(\mathbb{R}) \to \mathbb{R}$ by $U\big(p(x)\big) = p(0) + p'(0) + p''(0)$.

a. Calculate $U \circ T(p(x))$ (without using matrices).

b. Assuming the standard bases for $P_2(\mathbb{R})$ and $\mathbb{R}$ find matrix representations of $T,\ U,$ and $U \circ T$.

c. Show through matrix multiplication that the matrix representation of $U \circ T$ equals the product of the matrix representations of U and T.

8. Find two 2×2 matrices A and B such that $(A)(B) = 0$ (the zero matrix) but $A \neq 0$ and $B \neq 0$.