The Matrix Representation of a Linear Transformation- HW Problems

In Problems 1-5, using the standard basis for $\mathbb{R}^n$, $n \geq 1$, find a matrix representation for the following linear transformations.

1. $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $T(x_1, x_2) = (x_1 - x_2,\ 2x_1 + x_2)$
2. $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ by $T(x_1, x_2) = (x_1 - 2x_2,\ (3x_1 + x_2),\ x_2)$
3. $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ by $T(x_1, x_{2,} x_3) = (x_1 - 2x_2 + x_3,\ x_2 + x_3)$
4. $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(x_1, x_2, x_3) = (2x_1 - x_3,\ (-x_1 + 2x_2 + x_3),\ x_1)$
5. $T: \mathbb{R}^3 \rightarrow \mathbb{R}$ by $T(x_1, x_{2,} x_3) = x_1 - 2x_2 + 3x_3$

6. Let $\{e_1, e_2\}$ be the standard basis for $\mathbb{R}^2$ and $w_1 =< 1, 2, 0 >$, $w_2 =< 1, 0, 2 >$, $w_3 =< 0, 1, 2 >$ be vectors in $\mathbb{R}^3$ (written with respect to the standard basis for $\mathbb{R}^3$). Suppose $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ by $T(x_1, x_2) = x_1 w_1 + (x_1 - x_2) w_2 + (x_1 + x_2) w_3$.

a. Find a matrix representation of T with respect to the ordered bases $\{e_1, e_2\}$ and $\{w_1, w_2, w_3\}$.

b. Find a matrix representation of T with respect to the standard basis for $\mathbb{R}^2$ and $\mathbb{R}^3$.

c. Find a matrix representation of T with respect to the basis $\{< 1,1 >, < 2,3 >\}$ for $\mathbb{R}^2$ and $\{w_1, w_2, w_3\}$ for $\mathbb{R}^3$.

7. Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T(x_1, x_{2,} x_3) = (x_1 + x_2 + 2x_3,\ 2x_1 - x_2 - x_3)$$

with respect to the standard bases for $\mathbb{R}^3$ and $\mathbb{R}^2$.

a. Find a matrix representation of T with respect to the standard bases.

b. Let $w_1 =< 1, -1 >$ and $w_2 =< 1, 5 >$ be an ordered basis for $\mathbb{R}^2$. Find a matrix representation of T with respect to the basis $\{w_1, w_2\}$ for $\mathbb{R}^2$ and the standard basis for $\mathbb{R}^3$.

c. Let $v_1 =< 1, 0, 1 >$, $v_2 =< 0, 1, 1 >$, $v_3 =< 1, 1, 0 >$ be an ordered basis for $\mathbb{R}^3$. Find a matrix representation of T with respect to the basis $\{v_1, v_2, v_3\}$ for $\mathbb{R}^3$ and the standard basis for $\mathbb{R}^2$.

d. Find a matrix representation of T with respect to the basis $\{v_1, v_2, v_3\}$ for $\mathbb{R}^3$ (in part c) and the basis $\{w_1, w_2\}$ for $\mathbb{R}^2$ (in part b).

8. Let $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ by $T\big(p(x)\big) = p'(x) + p(x)$ where $p(x) = a_0 + a_1 x + a_2 x^2$ Find a matrix representation of T with respect to the standard basis for $P_2(\mathbb{R})$.

9. Let $T: M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ by $T(A) = (-A)^t$. Find a matrix representation for T with respect to the standard basis for $M_{2\times 2}(\mathbb{R})$,
$\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\}$.

10. Let $T: P_2(\mathbb{R}) \to \mathbb{R}^2$ by $T\big(p(x)\big) = (\int_0^1 p(x)dx,\ p'(1))$. Find a matrix representation of T with respect to the standard bases for $P_2(\mathbb{R})$ and $\mathbb{R}^2$.