Basis and Dimension- HW Problems

In problems 1-4 determine if the vectors form a basis for the given vector space. Explain your answer.

1. $< 1, 2, 3, >, \; < -2, 1, 4 >$ for $\mathbb{R}^3$
2. $< 1, 0, 1 >, \; < 0, 1, 1 >, \; < 2, 0, 1 >$ for $\mathbb{R}^3$
3. $< 1, 0, 1 >, \; < 0, 1, 1 >, \; < 2, 0, 1 >, \; < -1, 3, 1 >$ for $\mathbb{R}^3$
4. $x + 1, \quad x^2 + 1, \quad x^2 + x + 1$ for $P_2(\mathbb{R})$.

5. The vectors $v_1 =< 0, 2, 1 >, \; v_2 =< 1, 1, 1 >, \; v_3 =< 1, 2, 3 >,$ $v_4 =< -2, -4, 2 >,$ and $v_5 =< 3, -2, 2 >$ generate $\mathbb{R}^3$ (you can assume this). Find a subset of $\{v_1, v_2, v_3, v_4, v_5\}$ that forms a basis for $\mathbb{R}^3$.

6. Let $v_1 =< 2, -1, 3 >$ and $v_2 =< -1, 3, 1 >$ be vectors in $\mathbb{R}^3$. Find a vector $v_3 \in \mathbb{R}^3$ such that $S = \{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$. Show that S is a basis for $\mathbb{R}^3$.

7. $v_1 =< 1, 0, 0 >, \; v_2 =< 1, 1, 0 >$ and $v_3 =< 1, 1, 1 >$ is a basis for $\mathbb{R}^3$ (you can assume this). Given an arbitrary vector $w =< a, b, c >$ write w as a linear combination of $v_1, v_2,$ and v_3.

8. Find the dimension of the space spanned by

a. $x, \quad x-1, \quad x^2-1$ in $P_2(\mathbb{R})$

b. $x^3+2x^2+2, \quad x^3-3x+3, \quad x^3+6x^2+6x,$ in $P_3(\mathbb{R})$

In problems 9-13 find a basis and the dimension of W.

9. $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | \, x_1 + x_2 + x_3 = 0\}$

10. $W = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 | \, x_2 = x_3 \; and \; x_1 + x_4 = 0\}$

11. $W = \{A \in M_{3\times3}(\mathbb{R}) |$ A is an upper triangular matrix$\}$

12. $W = \{f \in P_3(\mathbb{R}) | \, f(0) = 0\}$.