Linear Systems and Linear Combinations- HW Problems

In problems 1-4 solve the linear systems via the method described in class.

1. $x_1 + 2x_2 - 3x_3 + x_4 = 1$

$-x_1 - x_2 + 4x_3 - x_4 = 6$

$-2x_1 - 4x_2 + 7x_3 - x_4 = 1$

2. $x_1 + 3x_2 + x_3 + x_4 = 3$

$2x_1 - 2x_2 + x_3 + 2x_4 = 8$

$x_1 - 5x_2 + x_4 = 5$

3. $x_1 + 2x_2 - 3x_3 + 4x_4 = 2$

$2x_1 + 5x_2 - 2x_3 + x_4 = 1$

$5x_1 + 12x_2 - 7x_3 + 6x_4 = 3$

4. $x_1 + 2x_2 - 3x_3 + 2x_4 = 2$

$2x_1 + 5x_2 - 8x_3 + 6x_4 = 5$

$3x_1 + 4x_2 - 5x_3 + 2x_4 = 4$

In problems 5 and 6 determine if the first vector is a linear combination of the other two vectors.

5. $< 1, 1, 3 >$, $< 2, -1, 3, >$, $< -1, 1, -1 >$ in $\mathbb{R}^3$

6. $2x^2 + 2x + 1$, $\; - x^2 + 2x + 1$, $\; - 2x^2 + 2x + 1$ in $P_2(\mathbb{R})$.

In problems 7-9 determine if the first vector is in the span of V.

7. $\begin{bmatrix} 5 & 4 \\ 5 & -2 \end{bmatrix}$; $\quad V = \{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\}$

8. $x^3 + 2x^2 - 3$; $\quad V = \{x^3 + x + 1,\; x^3 + x^2,\; x^2 + x + 1\}$

9. $< 2, -2, -3, 1 >$; $\quad V = \{< 1, 0, -1, 1 >,\; < 1, 1, 0, 1 >\}$

10. Show that $< 1, 0, -1 >$, $< -1, 1, 0 >$, $< 0, 1, 1 >$ generate $\mathbb{R}^3$.

11. Show that $2x^2$, $\; x^2 + 2x - 1$, $\; - x^2 + x + 1$ generate $P_2(\mathbb{R})$.

12. Show that $A_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $A_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $A_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ generate $M_{2\times 2}(\mathbb{R})$.