Jordan Canonical Form- HW Problems

In problems 1-5 the matrix representation A of a linear operator $T: V \to V$ is given with respect to a basis B. Find a new basis B' such that the matrix representation of T in the basis B' is in Jordan canonical form. Show by calculating $P^{-1}AP$, where P is the change of basis matrix from B to B', that the new matrix is in Jordan canonical form.

1. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$; If $P = \begin{bmatrix} -4 & 0 & 1 \\ 4 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}$ then

$$P^{-1} = \frac{1}{8}\begin{bmatrix} -1 & 1 & 0 \\ 3 & -3 & 8 \\ 4 & 4 & 0 \end{bmatrix}$$

3. $A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix}$; If $P = \begin{bmatrix} -1 & -1 & 1 \\ 2 & 3 & -2 \\ 1 & 1 & 0 \end{bmatrix}$ then

$$P^{-1} = \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

4. $A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ If $P = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ then

$$P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Hint: Show the eigenspace for $\lambda = 2$ is spanned by $< -1, 0, 1 >$ and $< 2, 1, 0 >$. To find the third vector solve $(A - 2I)v =< -1, 0, 1 >$.