Subspaces- HW Problems

In problems 1-5 determine if W is a subspace of $\mathbb{R}^3$ under the usual addition and scalar multiplication. Either show that it is or explain why it isn't.

1. $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | \; x_2 = 0\}$

2. $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | \; x_1 = x_2 \quad and \quad x_2 = 2x_3\}$

3. $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | \; x_1 + x_2 + 2x_3 = 1\}$.

4. $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | \; x_1 + x_2 + 2x_3 = 0\}$.

5. $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | \; x_2 = x_1^2\}$

In problems 6-8 Let $C(\mathbb{R})$ be the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ with the usual addition and scalar multiplication. Determine if W is a subspace of $C(\mathbb{R})$. Explain your answers.

6. $W = C^n(\mathbb{R}) = \{f \in C(\mathbb{R}) | \; f$ has a continuous n^{th} derivative$\}$

7. $W = \{f \in C^2(\mathbb{R}) | \; f''(x) + f(x) = 0\}$

8. $W = \{f \in C(\mathbb{R}) | \; f(-x) = f(x)\}$.

In problems 9-13 determine which subsets are subspaces of $M_{2\times 2}(\mathbb{R})$.

9. $W = \{A \in M_{2\times 2}(\mathbb{R}) |\ a_{12} = -a_{21}\}$

10. $W = \{A \in M_{2\times 2}(\mathbb{R}) |\ a_{12} = 0\}$

11. $W = \{A \in M_{2\times 2}(\mathbb{R}) |\ a_{12} = 1\}$

12. $W = \{A \in M_{2\times 2}(\mathbb{R}) |\ \det(A) = 1\}$

13. Fix $B \in M_{2\times 2}(\mathbb{R})$. Let $W = \{A \in M_{2\times 2}(\mathbb{R}) |\ AB = BA\}$.

In problems 14-16 determine whether the sets are subspaces of $P(\mathbb{R}) =$ {all polynomials with real coefficients}.

14. $W =$ {all polynomials with even degree}

15. $W =$ {all polynomials of degree 5}

16. $W =$ {all polynomials, $p(x)$, such that $p(0) = 0$}.