Eigenvalues and Eigenvectors- HW Problems

In problems 1-4 determine if the vector given is an eigenvector of the matrix A. If it is, find the eigenvalue associated with the vector.

1. $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$; $v =< 2, 2 >$

2. $A = \begin{bmatrix} 6 & -4 \\ 3 & -1 \end{bmatrix}$; $v =< 4, 3 >$

3. $A = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$; $v =< 1, 2 >$

4. $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{bmatrix}$; $v =< 1, 1, 1 >$.

In problems 5-9 given the matrix $A \in M_{n\times n}(\mathbb{R})$

a. Find all eigenvalues

b. For each eigenvalue λ of A find the eigenvectors corresponding to λ

c. If possible, identify a basis of $\mathbb{R}^n$ consisting of eigenvectors

d. If you can find a basis of eigenvectors, find an invertible matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix, and calculate $P^{-1}AP$.

5. $A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$

6. $A = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$

7. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$; You can assume if $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix}$ then $P^{-1} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

8. $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$; You can assume if $P = \begin{bmatrix} 1 & 3 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ then $P^{-1} = \begin{bmatrix} -1 & 3 & -1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$

9. $A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$.

In problems 10-13 let $T: V \to V$ be a linear transformation (ie a linear operator). Find the eigenvalues of T and an ordered basis for V so that the matrix of T with respect to that basis is diagonal.

10. $V = \mathbb{R}^2$; $T(x_1, x_2) = (x_1 + x_2,\ 4x_1 + x_2)$

11. $V = \mathbb{R}^3$; $T(x_1, x_2, x_3) = (x_1 + x_2,\ 2x_2 + 2x_3,\ 3x_3)$

12. $V = P_2(\mathbb{R})$;

$T(a_0 + a_1x + a_2x^2) = (a_0 + 2a_1 + a_2) + (3a_1 + a_2)x + (5a_1 - a_2)x^2$

13. $V = M_{2\times2}(\mathbb{R})$; $T(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

14. Let $A \in M_{2\times 2}(\mathbb{R})$. A is called idempotent if $A^2 = A$. Show that if λ is an eigenvalue of A, and A is idempotent, then $\lambda = 0$ or 1.

Hint: $\lambda v = Av = A(Av)$.