Solving Systems of Linear Equations Using Linear Transformations-
HW Problems

In problems 1-5 find a basis for the solution set of the homogeneous linear systems.

1.
$$\begin{aligned} x_1 + 2x_2 &= 0 \\ -2x_1 - 4x_2 &= 0 \end{aligned}$$

2.
$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_1 - x_2 - x_3 &= 0 \end{aligned}$$

3.
$$\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 0 \\ 2x_1 - 2x_2 + x_3 + 2x_4 &= 0 \\ x_1 - 5x_2 + \quad\quad x_4 &= 0 \end{aligned}$$

4.
$$x_1 + x_2 + x_3 + x_4 = 0$$

5.
$$\begin{aligned} x_1 + 2x_2 - 2x_3 + x_4 &= 0 \\ x_1 - 2x_2 + 2x_3 + x_4 &= 0 \end{aligned}$$

For problems 6-10 use your solutions to problems 1-5 to find all solutions to the following linear systems.

6.
$$x_1 + 2x_2 = 5$$
$$-2x_1 - 4x_2 = -10$$

7.
$$x_1 + x_2 + x_3 = 3$$
$$x_1 - x_2 - x_3 = -1$$

8.
$$x_1 + 3x_2 + x_3 + x_4 = 1$$
$$2x_1 - 2x_2 + x_3 + 2x_4 = -3$$
$$x_1 - 5x_2 + x_4 = -4$$

9.
$$x_1 + x_2 + x_3 + x_4 = 3$$

10.
$$x_1 + 2x_2 - 2x_3 + x_4 = -2$$
$$x_1 - 2x_2 + 2x_3 + x_4 = 10$$

For problems 11 and 12 write the system as $Ax = b$ and solve the system by finding A^{-1}.

11. $$2x_1 + 5x_2 = 3$$
$$x_1 + 3x_2 = 2$$

12. $$x_1 + \qquad\quad x_3 = 1$$
$$3x_1 + 3x_2 + 4x_3 = 2$$
$$2x_1 + 2x_2 + 3x_3 = 1$$

13. Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ by $T(x_1, x_2, x_3) = (x_1 + x_3,\ 2x_1 - x_2)$. Find $T^{-1}(8, 1)$.

14. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ by

$$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3,\ \ x_1 - x_2 + 3x_3,\ \ x_1 + 2x_3).$$

a. Is $< 6,\ 4,\ 5 > \in R(T)$?

b. Is $< -4,\ 2,\ 3 > \in R(T)$?

15. Suppose that $Ax = b$ is a system of n equations in n unknowns. Prove that if $Rank(A) = n$ then $Ax = b$ has a unique solution for every $b \in \mathbb{R}^n$.