Vector Spaces- HW Problems

In problems 1-7 determine whether the set with the definition of addition of vectors and scalar multiplication is a vector space. If it is, demonstrate that it is closed under addition and scalar multiplication and satisfies the 8 vector space axioms. If it's not, identify which items are violated. Assume the usual addition and scalar multiplication if it's not defined.

1. $V = \mathbb{R}^2, \quad < x_1, x_2 > + < y_1, y_2 > = < x_1 + x_2,\ y_1 + y_2 >$

$$c < x_1, x_2 > = < cx_1, x_2 >$$

2. $V = \mathbb{R}^2, \quad < x_1, x_2 > + < y_1, y_2 > = < x_1 + y_1,\ 0 >$

$$c < x_1, x_2 > = < cx_1, cx_2 >$$

3. $V = \{$all polynomials with real coefficients with degree ≥ 3 and the zero polynomial$\}$

4. $V = \{$all polynomials with real coefficients with only even powers of $x\}$ Note: this includes constants, ie $a_0 x^0$.

5. $V = \{f : \mathbb{R} \to \mathbb{R} \mid f(1) = 0\}$

6. $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0\}$

7. $V = \mathbb{R}, \quad x + y = \max(x, y), \quad cx = (c)(x)$ (usual multiplication)