Differentiable Maps Between Manifolds- HW Problems

1. Let f map the unit sphere into $\mathbb{R}$ by $f(x, y, z) = x^2$. Using the coordinate systems on S^2 given by

   $(u, v) = \pi_N(x, y, z) = (\frac{x}{1-z}, \frac{y}{1-z})$ on $S^2 - (0,0,1)$

   $(\bar{u}, \bar{v}) = \pi_S(x, y, z) = (\frac{x}{1+z}, \frac{y}{1+z})$ on $S^2 - (0,0,-1)$

   a. Find $\frac{\partial f}{\partial u}, \frac{\partial f}{\partial v}, \frac{\partial f}{\partial \bar{u}}, \frac{\partial f}{\partial \bar{v}}$.

   b. Find formulas relating $\frac{\partial f}{\partial u}$ to $\frac{\partial f}{\partial \bar{u}}$ and $\frac{\partial f}{\partial \bar{v}}$, as well as $\frac{\partial f}{\partial v}$ to $\frac{\partial f}{\partial \bar{u}}$ and $\frac{\partial f}{\partial \bar{v}}$.

   c. Consider the point on the sphere in cartesian coordinates given by $(-\frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2})$. Find the corresponding coordinates in (u, v) and then in $(\bar{u}, \bar{v})$.

   d. Verify that the relationships you found in parts a and b hold for this point.