

Connections and Covariant Differentiation- HW Problems

1. Let S^2 be parametrized by

$$\vec{\Phi}(x^1, x^2) = (\cos x^1 \sin x^2, \sin x^1 \sin x^2, \cos x^2), \quad \text{with the induced}$$

$$\text{metric } g = \begin{pmatrix} \sin^2 x^2 & 0 \\ 0 & 1 \end{pmatrix}.$$

- a. Find the 8 Christoffel symbols for the metric g .
 b. Let V be a vector field on S^2 given by

$$V = (x^1 x^2) \vec{\Phi}_{x^1} + x^1 \vec{\Phi}_{x^2} = \langle x^1 x^2, x^1 \rangle.$$

Find the components of ∇V .

2. Let S be the portion of a cone in \mathbb{R}^3 parametrized by

$$\vec{\Phi}(x^1, x^2) = (x^1 \cos x^2, x^1 \sin x^2, x^1); \quad x^1, x^2 \in \mathbb{R}, x^1 > 0$$

with the induced metric

$$g = \begin{pmatrix} 2 & 0 \\ 0 & (x^1)^2 \end{pmatrix}.$$

- a. Find the 8 Christoffel symbols for the metric g .
 b. Let V be a vector field on S^2 given by

$$V = (x^2)^2 \vec{\Phi}_{x^1} + (x^1)^2 (x^2) \vec{\Phi}_{x^2} = \langle (x^2)^2, (x^1)^2 (x^2) \rangle.$$

Find the components of ∇V .

3. Let $S = \mathbb{R}_+^2 = \{(x^1, x^2) \in \mathbb{R}^2 \mid x^2 > 0\}$ with the metric

$$g = \frac{1}{(x^2)^2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

- a. Find the 8 Christoffel symbols for the metric g .
- b. Let V be a vector field on S given by

$$V = \left\langle \frac{x^1}{x^2}, x^1 \right\rangle.$$

Find the components of ∇V .