Smooth Maps, Tangent Planes and Derivatives- HW Problems

1. Find an equation of the tangent plane to the hemisphere given by

   $$\vec{\Phi}(u,v) = (u, v, \sqrt{1-u^2-v^2})\ ;\ \ u^2+v^2 < 1,$$

   at the point where $u = \frac{1}{2}$ and $v = \frac{1}{2}$.

2. Find an equation of the tangent plane to the helicoid given by

   $\vec{\Phi}(u,v) = (v\cos(u), v\sin(u), 2u)$, at the point $(\sqrt{2}, \sqrt{2}, \frac{\pi}{2})$.

3. Suppose S is the upper hemisphere of the unit sphere given by

   $$\vec{\Phi}(u,v) = (u, v, \sqrt{1-u^2-v^2})\ ;\ \ u^2+v^2 < 1.$$

   Let $f: S \to S$ by

   $f\left(\vec{\Phi}(u,v)\right) = (-v, -u, \sqrt{1-u^2-v^2})$. Find $D_p f$.

4. Let S be the portion of the unit sphere given by

   $$\vec{\Phi}(u,v) = ((\cos(v))(\sin(u)), (\sin(v))(\sin(u)), \cos(u))$$

   where $0 \le u < \frac{\pi}{2}$, $0 < v \le 2\pi$. (ie the upper hemisphere).

   Let M also be the upper hemisphere but parametrized by

   $$\vec{\Psi}(u,v) = (\bar{u}, \bar{v}, \sqrt{1-\bar{u}^2-\bar{v}^2});\quad (\bar{u})^2 + (\bar{v})^2 < 1.$$

   Let $f: S \to M$ by $f(x,y,z) = (-y, x, z)$; $(x,y,z) \in S$.

   Find $D_p f$.

5. Let S be a surface in $\mathbb{R}^3$ given by $\vec{\Phi}(u,v) = (u, v,\ uv);\ \ u, v \in \mathbb{R}$.

Let $M = S_+^2 = \{(x,y,z) \in \mathbb{R}^3 |\ x^2 + y^2 + z^2 = 1,\ z > 0\}$ be the upper unit hemisphere. Define $f: S \to M$ by

$$f(u,v,uv) = \left(-\frac{v}{\sqrt{1+u^2+v^2}}, -\frac{u}{\sqrt{1+u^2+v^2}}, \frac{1}{\sqrt{1+u^2+v^2}}\right).$$

Let $\vec{\Psi}^{-1}: M \to \mathbb{R}^2$ by $\vec{\Psi}^{-1}\left(\bar{u}, \bar{v}, \sqrt{1-\bar{u}^2-\bar{v}^2}\right) = (\bar{u}, \bar{v})$.

a. Find $D_p f$ in the basis $\frac{\partial\vec{\Phi}}{\partial u}, \frac{\partial\vec{\Phi}}{\partial v}$.

b. Let $\vec{w} \in T_{(2,-2,-4)}S$, where $\vec{w} = \frac{\partial\vec{\Phi}}{\partial u} - 2\frac{\partial\vec{\Phi}}{\partial v}$ at the point where $u = 2$ and $v = -2$. Find $D_{(2,-2,-4)}f(\vec{w})$ in terms of the basis vectors $\frac{\partial\vec{\Psi}}{\partial\bar{u}}, \frac{\partial\vec{\Psi}}{\partial\bar{v}}$ and in terms of the standard basis in $\mathbb{R}^3$ and show that These two representations of $D_{(2,-2,-4)}f(\vec{w})$ are the same.