

Surfaces in \mathbb{R}^3 - HW Problems

1. Prove that the following functions provide parametrizations of the hyperbolic paraboloid $z = x^2 - y^2$.

a. $\vec{\Phi}(u, v) = (u + v, u - v, 4uv)$

b. $\vec{\Phi}(u, v) = (uv, u(1 - v), u^2(2v - 1))$

2. Find a parametrization for the following surfaces:

a. $z = y^3 - 3yx^2$ (called a "monkey saddle")

b. $y - x^2 - z^2 = 4$

c. $x - yz = 1$.

3. Show that:

a. $\vec{\Phi}(u, v) = (a(\cosh(u)) \cos(v), b(\cosh(u)) \sin(v), c(\sinh(u)))$,

is a parametrization of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ (called an elliptic hyperboloid of one sheet).

b. $\vec{\Phi}(u, v) = (a(\sinh(u)) \cos(v), b(\sinh(u)) \sin(v), c(\cosh(u)))$,

is a parametrization of $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (called an elliptic hyperboloid of two sheets).