Space Curves and the Frenet Formulas- HW Problems

1. Find the curvature, $\kappa(t)$, and the torsion, $\tau(t)$, of $\gamma(t) = (t - \sin(t), 1 - \cos(t), t)$.

2. Calculate the torsion, $\tau(t)$, of $\gamma(t) = (\cosh(t), \sinh(t), t)$.

3. Calculate the curvature, $\kappa(t)$, and the torsion, $\tau(t)$, and the Frenet frame $\vec{T}$, $\vec{N}$, and $\vec{B}$ for the curve $\gamma(t) = (2t,\ t^2,\ \frac{t^3}{3})$.

4. Let γ be a curve in $\mathbb{R}^3$ with $\gamma'(0) = (1,1,1)$, $\gamma''(0) = (2,0,1)$, and $\gamma'''(0) = (0,-1,1)$. Calculate $s'(0), s''(0)$, $\kappa(0), \tau(0)$, the Frenet frame $\vec{T}$, $\vec{N}$, and $\vec{B}$, and $\frac{d\vec{T}}{dt}, \frac{d\vec{N}}{dt}$, and $\frac{d\vec{B}}{dt}$ at $t = 0$, where $s(t)$ is the arc length function.

5. Let $\gamma(t) = (x(t),\ a,\ z(t))$; $a \in \mathbb{R}$, be a smooth, regular curve. Show by direct calculation that $\tau(t) = 0$ for all t as long as $\|\gamma'(t) \times \gamma''(t)\| \neq 0$.