# Normal Curvature and Geodesic Curvature- HW Problems

1. Take the elliptic paraboloid given by

$$\overrightarrow{\Phi}(u,v) = \left(u,\ v,\ \tfrac{1}{2}u^2 + \tfrac{1}{2}v^2\right);\quad (u,v) \in \mathbb{R}^2.$$

   Now consider the curve $u(t) = t^2$, $v(t) = t$ and its image under $\overrightarrow{\Phi}$ on this surface. At the point where $t = 1$ calculate the geodesic curvature, $\kappa_g$, the normal curvature, $\kappa_n$, and the curvature $\kappa$. From that calculation show that $\kappa^2 = \kappa_n^2 + \kappa_g^2$.

2. Let $S$ be a surface given by

$$\overrightarrow{\Phi}(u,v) = \left(u,\ v,\ \tfrac{1}{2}u^2 + \tfrac{1}{3}v^3\right);\quad (u,v) \in \mathbb{R}^2.$$

   Let $\gamma$ be a curve on $S$ given by the image of $\big(u(t), v(t)\big) = (t, 0)$ under $\overrightarrow{\Phi}$. Find the normal curvature, $\kappa_n$, the geodesic curvature, $\kappa_g$, and the curvature, $\kappa$, of $\gamma$.

3. Show that if $\gamma(t)$ is a curve in the $xy$ plane, i.e., $\gamma(t) = (u(t), v(t), 0)$, then the general formula for the geodesic curvature:

$$\kappa_g = \frac{\gamma''(t)\cdot\left(\vec{N}\times\gamma'(t)\right)}{\left(E\left(\frac{du}{dt}\right)^2 + 2F\left(\frac{du}{dt}\right)\left(\frac{dv}{dt}\right) + G\left(\frac{dv}{dt}\right)^2\right)^{\frac{3}{2}}}$$

becomes in this case:

$$\kappa_g = \frac{u'v'' - v'u''}{((u')^2 + (v')^2)^{\frac{3}{2}}} = \text{signed curvature } \kappa_s.$$

In addition, for a plane curve $\kappa_n = 0$.

Hint: Calculate the first and second fundamental forms of the $xy$ plane by using $\vec{\Phi}(u, v) = (u, v, 0)$.