

## The Second Fundamental Form- HW Problems

1. Calculate the second fundamental form for the following parametrization of the upper hemisphere:

$$\vec{\Phi}(u, v) = (u, v, \sqrt{1 - u^2 - v^2}); \quad u^2 + v^2 < 1.$$

2. For the following surfaces calculate the second fundamental form and identify all points that are elliptic, hyperbolic, parabolic, or planar.

a.  $\vec{\Phi}(u, v) = ((\cos(u)) \sin(v), (\sin(u)) \sin(v), 3(\cos(v)));$

where  $(u, v) \in [0, 2\pi) \times [0, \pi)$ . (Ellipsoid)

b.  $\vec{\Phi}(u, v) = ((\cosh(v)) \cos(u), (\cosh(v)) \sin(u), v)$

where  $(u, v) \in [0, 2\pi) \times \mathbb{R}$ . (Catenoid)

c.  $\vec{\Phi}(u, v) = \left(u, v, \frac{1}{2}u^2 + \frac{1}{3}v^3\right); \quad \text{where } (u, v) \in \mathbb{R}^2$

d.  $\vec{\Phi}(u, v) = (v \cos(u), v \sin(u), u); \quad \text{where } (u, v) \in [0, 2\pi] \times \mathbb{R}$

(Helicoid).