

Level Surfaces and Quadric Surfaces- HW Problems

1. Prove that the following surfaces are smooth surfaces:

- a. $\frac{x^2}{p^2} + \frac{y^2}{q^2} + \frac{z^2}{r^2} = 1$; (ellipsoid)
- b. $z = \frac{x^2}{p^2} - \frac{y^2}{q^2}$; (hyperbolic paraboloid)
- c. $\frac{x^2}{p^2} + \frac{y^2}{q^2} = 1$ (elliptic cylinder)
- d. $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^4 = 1\}$

2. Identify the following quadric surfaces by putting the equations in standard form:

- a. $x^2 - y^2 - 2x - z + 1 = 0$
- b. $\vec{\Phi}(u, v) = (u \sin(2v), u, u \cos(2v))$
- c. $\vec{\Phi}(u, v) = (2 \sin(u), v, 2 \cos(u))$
- d. $\vec{\Phi}(u, v) = (\sinh^2 u, (\sinh(u)) \sinh(v), (\sinh(u)) \cosh(v)).$