

Curves in \mathbb{R}^2 and \mathbb{R}^3 /Arc Length- HW Problems

1. Find parametrizations for the following curves.
 - a. $4x^2 + y^2 = 1$
 - b. $y = e^x$
 - c. $3x + 4y = 1$.

2. Find cartesian equations (i.e., an equation in x and y) for the following curves:
 - a. $\gamma(t) = (\cos^3 t, \sin^3 t); \quad 0 \leq t \leq 2\pi$
 - b. $\gamma(t) = (\sinh(t), t); \quad t \in \mathbb{R}$. (recall that $\sinh(t) = \frac{e^t - e^{-t}}{2}$)

3. Calculate the tangent/velocity vectors at the given point for the following curves:
 - a. $\gamma(t) = (t^2 + 1, t, e^{3t})$ at $t = 0$
 - b. $\gamma(t) = (t^2 - 1, t, \cos(\pi t))$ at $(0, 1, -1)$.

4. A particle moves along a helix given by
$$\gamma(t) = (\cos(3t), \sin(3t), 4t).$$
 - a. Find the speed of the particle at $t = \frac{\pi}{3}$.
 - b. Find the speed of the particle at any time t .

5. Show that $\gamma(t) = (2 + \frac{2}{5}\sin(2t), \frac{1}{2}\cos(2t), \frac{3}{10}\sin(2t))$ is a unit speed curve and find its length between $t = 3$ and $t = 7$.

6. Find the length of the curve $\gamma(t) = (2t, t^2, \ln(t))$ between the points $(2,1,0)$ and $(4,4, \ln(2))$.