The Convolution Theorem/Derivatives & Integrals of Transforms-
HW Problems

In problems 1-4 calculate $f(t) * g(t)$.

1. $f(t) = 1, \quad g(t) = e^t$
2. $f(t) = t, \quad g(t) = \sin(t)$
3. $f(t) = t^2, \quad g(t) = e^t$
4. $f(t) = t, \quad g(t) = t$

5a. Show that if $f(t) = e^{at}$ and $g(t) = e^{bt}$, where a, b are constants, then

$$(f * g)(t) = \frac{1}{a-b}(e^{at} - e^{bt}).$$

b. By direct calculation of Laplace transforms, show that

$$\mathcal{L}\big((f * g)(t)\big) = (\mathcal{L}(f(t))(\mathcal{L}(g(t))$$

for $f(t) = e^{at}$ and $g(t) = e^{bt}$ (where $\mathcal{L}$ is the Laplace transform).

In problems 6-8 use the convolution theorem to find the inverse Laplace transform of the given function.

6. $F(s) = \frac{3}{s^2-1}$
7. $F(s) = \frac{1}{(s-1)^2}$
8. $F(s) = \frac{4}{s(s^2+4)}$

In problems 9-11 find the Laplace transform of the given function.

9. $f(t) = tcos(3t)$

10. $f(t) = t^2 \sin(3t)$

11. $f(t) = \frac{e^t - 1}{t}$

In problems 12-14 find the inverse Laplace transform of the given function.

12. $F(s) = \ln(\frac{s+3}{s-3})$

13. $F(s) = \ln(\frac{s^2+4}{s+4})$

14. $F(s) = \frac{2s}{(s^2+1)^2}$

15. Use the Laplace transform to transform $ty'' + (t-2)y' + y = 0$ to find a solution where $y(0) = 0$ but $y(t) \not\equiv 0$.