

Series Solutions near Ordinary Points- HW Problems

In problems 1-3 find a series solutions around 0 (ie in powers of x) for the differential equation using a recurrence relation. Find the guaranteed radius of convergence of the solution.

1. $y'' - xy' - y = 0$

2. $(x^2 + 1)y'' - 4xy' + 6y = 0$

3. $(x^2 - 3)y'' + 2xy' = 0$

In problems 4-6 solve the initial value problems with a power series.

4. $(1 + x^2)y'' + 2xy' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$

5. $y'' + xy' - 2y = 0, \quad y(0) = 0, \quad y'(0) = 3$

6. $y'' + 2xy' + 2y = 0, \quad y(0) = 6, \quad y'(0) = 2$

7. Solve the initial value problem by first making a substitution $t = x - a$ and then finding a series solution of the form $\sum_{n=0}^{\infty} c_n t^n$. Then transform the solution back to a power series in $x - a$.

$$y'' + (x - 1)y' + y = 0, \quad y(1) = 3, \quad y'(1) = -1.$$