Contour Integration- HW Problems

1. Evaluate $\int_C \bar{z}^2 dz$ where C is

a. $x^2 + y^2 = 1, \ \ y \leq 0.$

b. The line segment from i to $3 + 4i$.

c. The circle $|z - 1| = 1.$

d. The triangle with vertices at $0, 3,$ and $3 + 3i$.

e. The curve $y = 2x^2; \quad -1 \leq x \leq 0.$

2. Let C be the circle $x^2 + y^2 = 4$. Evaluate $\oint_C f(z)dz$ where

a. $f(z) = z^2 - 1$

b. $f(z) = z + \frac{1}{z}$

c. $f(z) = \frac{1}{\bar{z}}$

d. $f(z) = |z|$

e. $f(z) = Im(z)$

3. Let C be the square with vertices at $0, 2, 2 + 2i,$ and $2i$. Evaluate $\oint_C f(z)dz$ where $f(z) = \cos(z)$.

4. Let C be the open upper semicircle of radius $R > 1$ given by $x^2 + y^2 = R^2,\ \ y > 0$. Show that

a. $\left| \int_C \frac{dz}{z^3+1} \right| \leq \frac{\pi R}{R^3-1}$.

b. $\lim_{R\to\infty} \int_C \frac{dz}{z^3+1} = 0.$