

## Multivalued Functions- HW Problems

1. Which of the following functions is/are multivalued?

a.  $f(z) = \frac{1}{z^3}$

b.  $g(z) = z^{\frac{1}{5}}$

c.  $f(z) = \log(z)$

d.  $g(z) = e^z$

e.  $f(z) = \sinh(z)$

f.  $g(z) = \sinh^{-1}(z)$

2. Find all values of the following expressions. Identify the principal value in each case.

a.  $(1 + i)^{\frac{1}{3}}$

b.  $\log(1 + i)$

c.  $\cos^{-1}\left(\frac{1}{2}\right)$

d.  $i^i$  (Hint:  $i^i = (e^{\log(i)})^i = e^{i[\log(i)]}$ )

3. Find branch points for the following functions.

a.  $f(z) = (z - 2)^{\frac{1}{2}}$

b.  $g(z) = \log(z^2)$

4. Find all solutions of  $9 + 3e^{(2-z)} = 6$

5. Derive the following formulas:

a.  $\cos^{-1}(z) = -i \log[z + i(1 - z^2)^{\frac{1}{2}}]$

b.  $\tan^{-1}(z) = \frac{1}{2i} \log\left[\frac{i-z}{i+z}\right]$

6. Given that  $\cosh^{-1}(z) = \log[z + (z^2 - 1)^{\frac{1}{2}}]$  find  $\frac{d}{dz}(\cosh^{-1}(z))$ .