The Cauchy-Riemann Equations- HW Problems

In problems 1-3 show that the real and imaginafy parts of the functions satisfy the Cauchy-Riemann equations everywhere and that the functions are analytic everywhere.

1. $f(z) = \sin(z)$ (Hint: use $\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$)
2. $f(z) = \cosh(z)$
3. $f(z) = ze^z$

4. Show that $f(z) = x^2 + iy^2$ is not analytic anywhere even though the Cauchy-Riemann equations are satisfied at $(0,0)$.

5. Use the Cauchy-Riemann equations to determine which of the following functions is/are analytic on $\mathbb{C}$?

a. $f(z) = \bar{z}$

b. $g(z) = e^{(z^2)}$

c. $g(z) = e^{\bar{z}}$

6. Show that the following functions are harmonic and find all harmonic conjugates.

a. $u(x, y) = e^x \sin(y)$

b. $u(x, y) = x^2 - y^2$

c. $u(x, y) = (\sin(x))(\cosh(y))$

7. Suppose that $f(z)$ is analytic in a domain $D \subseteq \mathbb{C}$. Prove that if $\bar{f}(z)$ is analytic then $f(z)$ is a constant function.