The Derivative of a Complex Function- HW Problems

1. Where are the following functions analytic? Find the derivative where it exists.

a. $f(z) = \cosh(z)$

b. $g(z) = \csc(z)$

c. $h(z) = \sin\left(\frac{1}{z}\right)$

d. $f(z) = 5\bar{z}$

e. $g(z) = \frac{3z^2+2}{z^3-1}$

2. Use the limit definition of a derivative to show that $f(z) = \frac{z-\bar{z}}{2i} = Im(z)$ and $g(z) = \frac{z+\bar{z}}{2} = Re(z)$ are not differentiable for any value of z.

3. Use the limt definition of a derivative to show that $f(z) = \bar{z}^2$ is not analytic on any domain $D \subseteq \mathbb{C}$.