Laurent Series- HW Problems

For Problems 1-5 find the Taylor/Laurent series in powers of z in the region given. Make sure you include the n^{th} term.

1. $\frac{\sin(z)}{z^4}$; $0 < |z| < \infty$

2. $\frac{e^{(z^2)}-1-z^2}{z^7}$; $0 < |z| < \infty$

3a. $\frac{z}{1+z^2}$; $|z| < 1$

b. $\frac{z}{1+z^2}$; $|z| > 1$

4a. $\frac{z}{4-z^2}$; $|z| < 2$

b. $\frac{z}{4-z^2}$; $|z| > 2$

5a. $\frac{1}{(z-1)(z-2)}$; $|z| < 1$

b. $\frac{1}{(z-1)(z-2)}$; $1 < |z| < 2$

c. $\frac{1}{(z-1)(z-2)}$; $|z| > 2$

6. Find the first 4 non-zero terms of the Laurent series about the given point and then use that series to find the integral of the function around a unit circle centered at that point.

a. $f(z) = \frac{e^{3z}}{(z+1)^3}$; $z = -1$

b. $g(z) = (z-1)\sin(\frac{1}{z+3})$; $z = -3.$