

Taylor Series- HW Problems

In problems 1-5 find the radius of convergence of the series.

1.
$$\sum_{n=1}^{\infty} \frac{z^n}{3^{(n+2)}}$$

2.
$$\sum_{n=1}^{\infty} \frac{2^{2n} z^n}{n}$$

3.
$$\sum_{n=1}^{\infty} \frac{n z^{2n}}{2^n}$$

4.
$$\sum_{n=1}^{\infty} \frac{z^{3n}}{n!}$$

5.
$$\sum_{n=1}^{\infty} \frac{n^n}{n!} z^n$$

In problems 6-9 Find the Taylor series around $a = 0$.

6.
$$f(z) = \frac{1}{1+z} \quad \text{for } |z| < 1$$

7.
$$g(z) = \frac{z}{4+z^2} \quad \text{for } |z| < 2$$

8.
$$f(z) = \frac{\cos(z^2) - 1 + \frac{z^4}{2}}{z^3} \quad \text{for } 0 < |z|$$

9.
$$g(z) = \frac{e^{(z^3)} - 1 - z^3}{z^4} \quad \text{for } 0 < |z|.$$

In problems 10 and 11 evaluate $\oint_C f(z)dz$ where C is the circle $|z| = 1$.

$$10. \quad f(z) = \frac{e^z - 1 - z - \frac{z^2}{2}}{z^4}$$

$$11. \quad f(z) = \frac{\cos(z^2) - 1 + \frac{z^4}{2}}{z^7}.$$

12. Use the Taylor series for $\frac{1}{1-z}$ around $a = 0$ to find a series representation of $\text{Log}(1 - z)$ for $|z| < 1$.

13. Use the Taylor series for $\frac{1}{1+z}$ around $a = 0$ to find a series representation for $\frac{1}{(1+z)^2}$ (Hint: $\frac{d}{dz} \left(\frac{1}{1+z} \right) = -\frac{1}{(1+z)^2}$).