

## Integration- HW Problems

1. Let  $g: [0,1] \times [0,1] \rightarrow \mathbb{R}$  by

$$\begin{aligned} g(x, y) &= 1 && \text{if } 0 \leq x < \frac{1}{2} \\ &= 0 && \text{if } \frac{1}{2} \leq x \leq 1. \end{aligned}$$

Show that  $g$  is integrable and  $\int_{[0,1] \times [0,1]} g = \frac{1}{2}$ .

2. Let  $f, g: B \subseteq \mathbb{R} \rightarrow \mathbb{R}$  be integrable. Assuming that  $f(x) \leq g(x)$  for all  $x \in B$  show that  $\int_B f \leq \int_B g$ .

3. Suppose  $f: [0,1] \rightarrow \mathbb{R}$ , by

$$\begin{aligned} f(x) &= 1 && \text{if } x = \frac{1}{2} \\ &= 0 && \text{if } x \neq \frac{1}{2}. \end{aligned}$$

Prove that

- i.  $f$  is integrable
- ii.  $\int_{[0,1]} f = 0$ .

4. Find an example of a function  $f: [0,1] \rightarrow \mathbb{R}$  such that  $f$  is not integrable, but  $f^2$  is integrable.