

The First Fundamental Form of a Surface in \mathbb{R}^3 - HW Problems

1. Let The upper hemisphere of the unit sphere be parametrized by

$$\vec{\Phi}(u, v) = (u, v, \sqrt{1 - u^2 - v^2}), \quad u^2 + v^2 < 1.$$

- a. Find the first fundamental form/metric tensor of $\vec{\Phi}$.
- b. Use this metric tensor to find the length of the portion of the great circle starting at $(0,0,1)$ and ending at $(1,0,0)$.
- c. Using the metric tensor in part a, find the length of the curve on the upper hemisphere given by $x^2 + y^2 = \frac{1}{2}$, $z = \frac{\sqrt{2}}{2}$.