The Inverse Function Theorem and The Implicit Function Theorem-HW Problems

1. Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ and $f'(x)$ is continuous. Show that if $f'(x) \neq 0$ for all x then f is one-to-one on $\mathbb{R}$.

2. Let $g(x, y) = (e^x cosy, e^x siny)$. Show that $\det\big(Dg(x, y)\big) \neq 0$ for all $(x, y) \in \mathbb{R}^2$ but $g(x, y)$ is not one-to–one on $\mathbb{R}^2$.

3. Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$, where $f(s, t) = (s^3 - 3st^2,\ 3s^2t - t^3)$. Let $q = (2,3)$. Prove there exists a neighborhood, V, of q and a neighborhood, W, of $f(q)$ such that f has a continuously differentiable inverse. Find $Df^{-1}(-46, 9)$.

4. Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$, by $f(x, y) = (x^2e^y, xy)$.
   a. Find $Df(2,0)$.
   b. Find $Df^{-1}\big(f(2,0)\big)$.