

Integrating Differential Forms over Manifolds- HW Problems

1. Let T^2 be the torus embedded in \mathbb{R}^4 by

$$\vec{\Phi}(u, v) = (\cos u, \sin u, \cos v, \sin v); \quad (u, v) \in [0, 2\pi] \times [0, 2\pi].$$

Find $\int_{T^2} \omega$ where ω is given by

- a. $\omega = x_1^3 dx_1 \wedge dx_4$
- b. $\omega = x_1^3 x_3 dx_2 \wedge dx_4$
- c. $\omega = x_1 x_4 dx_2 \wedge dx_3$

2. Consider the torus $T^2 \subseteq \mathbb{R}^3$ given by

$$\vec{\Phi}(u, v) = ((3 + \cos v)\cos u, (3 + \cos v)\sin u, \sin v);$$

where $(u, v) \in [0, 2\pi] \times [0, 2\pi]$.

Let $\omega = -y dx \wedge dz + x dy \wedge dz$.

Evaluate $\int_{T^2} \omega$.