Representing Tangent Spaces on Manifolds- HW Problems

1. Using the parametrization of S^2 given by

$\vec{\Phi}(\varphi,\theta) = (cos\theta sin\varphi, sin\theta sin\varphi, cos\varphi), \quad 0 \le \varphi \le \pi,\ 0 \le \theta \le 2\pi,$

Find an equation of the tangent plane to S^2at $(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2})$. Use $D\vec{\Phi}$ to Find this tangent plane.

2. Let a 3-dimensional manifold in $\mathbb{R}^4$ be given by

$$M = \{(x_1, x_2, x, x_4) \in \mathbb{R}^4 |\ x_1 = x_2^2 - x_3^2 + x_4^2\}.$$

   a. Find 3 vectors that span T_pM, where $p = (6,3,2,1)$.
   b. Find an equation in $\mathbb{R}^4$ for the tangent space at p.

3. A 2 dimensional torus is a surface given by $S^1 \times S^1$. A 3 dimensional torus is a solid given by $S^1 \times S^1 \times S^1$. Below is a parametrization of a 3 dimensional torus in $\mathbb{R}^4$:

$\vec{\Phi}(u_1,u_2,u_3) = ((4 + (2 + cosu_1)cosu_2)cosu_3,$
$(4 + (2 + cosu_1)cosu_2)sinu_3, (2 + cosu_1)sinu_2, sinu_1)$

   where $(u_1,u_2,u_3) \in [0,2\pi] \times [0,2\pi] \times [0,2\pi]$.

   a. Find 3 vectors that span the tangent space at

$$\vec{\Phi}\left(0,0,\frac{\pi}{3}\right) = (\frac{7}{2}, \frac{7\sqrt{3}}{2}, 0, 0).$$

   b. Find an equation in $\mathbb{R}^4$ for this tangent space.

4. A 3 dimensional torus in $\mathbb{R}^6$ is given by

$$\vec{\Phi}(u_1, u_2, u_3) = (cosu_1, sinu_1, cosu_2, sinu_2, cosu_3, sinu_3)$$

where $(u_1, u_2, u_3) \in [0,2\pi] \times [0,2\pi] \times [0,2\pi]$.

a. Find 3 vectors in $\mathbb{R}^6$ that span the tangent space at

$$\vec{\Phi}\left(\frac{\pi}{4}, 0, \frac{\pi}{2}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1, 0, 0, 1\right).$$