Differentiable Maps Between Manifolds- HW Problems

1. Let f map the unit sphere into $\mathbb{R}$ by $f(x,y,z) = x^2$. Using the coordinate systems on S^2 given by

$$(u,v) = \pi_N(x,y,z) = (\frac{x}{1-z}, \frac{y}{1-z}) \quad \text{on} \quad S^2 - (0,0,1)$$
$$(\bar{u},\bar{v}) = \pi_S(x,y,z) = (\frac{x}{1+z}, \frac{-y}{1+z}) \quad \text{on} \quad S^2 - (0,0,-1):$$

a. Find $\frac{\partial f}{\partial u}, \frac{\partial f}{\partial v}, \frac{\partial f}{\partial \bar{u}}, \frac{\partial f}{\partial \bar{v}}$.

b. Find formulas relating $\frac{\partial f}{\partial u}$ to $\frac{\partial f}{\partial \bar{u}}$ and $\frac{\partial f}{\partial \bar{v}}$, as well as $\frac{\partial f}{\partial v}$ to $\frac{\partial f}{\partial \bar{u}}$ and $\frac{\partial f}{\partial \bar{v}}$.

c. Consider the point on S^2 in cartesian corrdinates given by $(-\frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2})$. Find the corresponding coordinates in (u,v) and $(\bar{u},\bar{v})$.

d. Verify that the relationships you found in parts a and b hold for this point.