

Stereographic Projections of Spheres- HW Problems

1. $\pi_1: S^2 - (0,0,1) \rightarrow \mathbb{R}^2$ by $\pi_1(x, y, z) = \left(\frac{x}{1-z}, \frac{y}{1-z}\right)$ is a stereographic projection of the unit sphere onto \mathbb{R}^2 . The inverse function $\pi_1^{-1}: \mathbb{R}^2 \rightarrow S^2 - (0,0,1)$ by

$\pi_1^{-1}(u, v) = \left(\frac{2u}{u^2+v^2+1}, \frac{2v}{u^2+v^2+1}, \frac{u^2+v^2-1}{u^2+v^2+1}\right)$ is a parametrization of a portion of the sphere S^2 . Find the first fundamental form of this parametrization.

2. One can find a different stereographic projection of a unit sphere by taking the sphere $x^2 + y^2 + (z - 1)^2 = 1$ and taking the line through the north pole $(0,0,2)$ and any point on the sphere (x, y, z) and finding the intersection with the x, y plane. Find a formula for this stereographic projection,

$\pi: S^2 - (0,0,2) \rightarrow \mathbb{R}^2$, and its inverse $\pi^{-1}: \mathbb{R}^2 \rightarrow S^2 - (0,0,2)$.