Integration over Singular n-Chains and Stokes' Theorem- HW Problems

1. In each case below calculate the value of the integral by calculating $\int_{[0,1]^k} c^*\omega$.
   a. $\iint_D \omega$ where $\omega = e^{x^2+y^2} dx \wedge dy$ and D is the disk of radius 2 centered at the origin.
   b. $\int_c \omega$ where $c(t) = (cos2\pi t, sin2\pi t, 2\pi t)$; $0 \leq t \leq 1$, and $\omega = (x^2 + y^2)dx + xdy + z^2dz$.
   c. $\iint_D \omega$ where $\omega = xdx \wedge dy + zdz \wedge dx$, and D is the upper unit hemisphere which is the image of $c: [0,1]^2 \to \mathbb{R}^3$, by $c(r, \theta) = (rcos2\pi\theta,\ rsin2\pi\theta,\ \sqrt{1-r^2})$.

2. Use Stokes' theorem and $\omega = -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$ to show the unit circle, c, is not the boundary of any $2-$chain in $\mathbb{R}^2 - (0,0)$.

3. Consider the 2-form $\omega = \frac{xdy\wedge dz+ydz\wedge dx+zdx\wedge dy}{(x^2+y^2+z^2)^{\frac{3}{2}}}$ on $\mathbb{R}^3 - (0,0,0)$.

   a. Show that $d\omega = 0$.
   b. Show that $\omega \neq d\eta$ for any 1-form η on $\mathbb{R}^3 - (0,0,0)$.
      Hint: integrate ω over the unit sphere. You can assume that $c: [0,1]^2 \to S^2 \subseteq R^3$ and $c^*\omega = 2\pi^2(sin\pi\varphi)d\varphi d\theta$. Make sure you explain why your argument works.