Calculations with the Metric Tensor- HW Problems

1. Let the surface $S \subseteq \mathbb{R}^3$ be parametrized by
$$\vec{\Phi}(u,v) = (u, v, u^2 - v^2), \quad u, v \in \mathbb{R}.$$

a. Find the metric tensor, g, at $(u,v) = (2,1)$.

b. If $\vec{w}_1 = 4\vec{\Phi}_u(2,1) - \vec{\Phi}_v(2,1)$ and
$\vec{w}_2 = \vec{\Phi}_u(2,1) + 2\vec{\Phi}_v(2,1)$,
find $g(\vec{w}_1, \vec{w}_2)$.

c. Find $\left(D\vec{\Phi}(2,1)\right)^* g(\vec{v}_1, \vec{v}_2)$, where $\vec{v}_1 =< 1,3 >$,
$\vec{v}_2 =< -2,1 >$, where $\vec{v}_1, \vec{v}_2 \in T_{(2,1)}\mathbb{R}^2$.