Functions on $\mathbb{R}^n$- HW Problems

1. Prove that $|a-b| \leq |a|+|b|$ for all $a, b \in \mathbb{R}$.

2. Prove that $||a|-|b|| \leq |a-b|$ for all $a, b \in \mathbb{R}$.

3. Let $f: A \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m$ by

$f(x_1, \dots, x_n) = (f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n))$. Give an ϵ/δ proof f is continuous at $a \in A$ if and only if $f_i(x),\ 1 \leq i \leq m$ is continuous at a.

4. Let $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation. Give an ϵ/δ proof that T is continuous at any point $a \in \mathbb{R}^n$. Hint: Use the fact that there exists an $M \in \mathbb{R}$ such that $|T(a)| \leq M|a|$ for all $a \in \mathbb{R}^n$.

5. Give an ϵ/δ proof that if $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$, then $\lim_{x \rightarrow a} |f(x)| = 0$ if and only if $\lim_{x \rightarrow a} |f_i(x)| = 0$, where $f(x) = \big(f_1(x), \dots, f_n(x)\big)$.

6. Let $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$. Give an ϵ/δ proof that if $f(x)$ is continuous at $x = a$ then $|f(x)|$ is continuous at Hint: Use $||x|-|y|| \leq |x-y|$.