The Chain Rule- HW Problems

1. Let $f(x,y,z) = xy + z^2$. Suppose $x = \rho\cos(\theta)\sin(\phi)$, $y = \rho\sin(\theta)\sin(\phi)$, $z = \rho\cos(\phi)$. Use the chain rule to find $\frac{\partial f}{\partial \rho}$, $\frac{\partial f}{\partial \theta}$, and $\frac{\partial f}{\partial \phi}$ in terms of $\rho, \theta,$ and ϕ.

2. Find $\frac{dz}{dt}$ using the chain rule.

a. $z = \cos(x + 4y)$; $x = 5t^2$, $y = \frac{1}{t}$.

b. $z = xe^y$; $x = t^2$, $y = 1 + \ln(t)$.

3. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ using the chain rule.

a. $z = x^2y^4$, $x = (s)cos(t)$, $y = (s)sin(t)$.

b. $z = \tan^{-1}(x - y)$, $x = s^2 + t^2$, $y = 1 - 2st$.

4. Use the chain rule to find the indicated partial derivatives.

a. $u = \sqrt{r^2 + s^2}$, $r = y + (x)\cos(t)$, $s = x + (y)\sin(t)$.
Find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial t}$, when $x = 1$, $y = 2$, $t = 0$.

b. $M = xe^{(y-z^2)}$, $x = 2uv$, $y = u - v$, $z = u + v$.
Find $\frac{\partial M}{\partial u}$, $\frac{\partial M}{\partial v}$ when $u = 3$, $v = -1$.

5. Let $g(x, y, z) = z^2 + zln(x^2 + y^2)$,
$x = 2s + t, \quad y = -2s + t, \quad z = 2st.$

Use the chain rule to find $\frac{\partial g}{\partial s}$, $\frac{\partial g}{\partial t}$, when $s = 1, \ t = 0.$