The Double Integral over a Rectangle- HW Problems

In problems 1-3, evaluate the iterated Integrals. First integrate with respect to x and then with respect to y. Then evaluate the integral by reversing the order of integration. That is, integrate first with respect to y and then integrate with respect to x.

1. $\int_0^1 \int_0^1 (6x^2y + 2x + 3y^2)dxdy$
2. $\int_0^{\pi} \int_0^{\frac{\pi}{2}} (\sin(x))(\cos(y))dxdy$
3. $\int_0^1 \int_1^2 (3x^2 - 4xy)dxdy$

4. Evaluate $\iint_R xe^y dydx$ where $R = [1,3] \times [0, \ln(2)]$.

5. Find the volume of the solid that lies over the rectangle $[1,2] \times [0,2]$ and is bounded above by the following functions.

a. $f(x,y) = 30 - 3x^2 - 3y^2$

b. $f(x,y) = 2 + 4x + 2y$

c. $f(x,y) = 3x^2 + 3y^2$.

6. Evaluate the following integrals using the property that

$$\int_c^d \int_a^b f(x)g(y)dxdy = (\int_c^d g(y)dy)(\int_a^b f(x)dx)$$

a. $\iint_R \frac{e^x}{y^2} dA$ where $R = [0, \ln(5)] \times [1, 2]$.

b. $\iint_R \frac{6x^2y}{y^2+1} dA$ where $R = [0, 2] \times [1, 3]$.

Evaluate the following integrals.

7. $\iint_R \cos(x + y)dA$ where $R = [0, 1] \times [0, 1]$.

8. $\iint_R \frac{2y}{1+x^2} dA$ where $R = [0, 1] \times [0, 2]$.

9. Calculate the volume of the solid that's bounded by $z = x^4 + y^2$, the x-y plane, and the planes $x = 0,\ x = 1,\ y = 0,\ y = 1$.

10. Calculate the volume of the solid bounded by the surface $z = e^x$, the planes $x = 0,\ x = 2,\ y = 0,\ y = \ln(3),$ and the x-y plane.