

Error Estimation Using Taylor Polynomials- HW Problems

Approximate the function $f(x)$ with a Taylor polynomial of the given degree around the given point a . Estimate the accuracy of the Taylor polynomial on the given interval.

1. $f(x) = \sqrt{x}$, $a = 9$, $n = 2$, $9 \leq x \leq 9.3$

2. $f(x) = \frac{1}{x^3}$, $a = 1$, $n = 2$, $0.9 \leq x \leq 1.1$

3. $f(x) = \cos(x)$, $a = \frac{\pi}{4}$, $n = 4$, $\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$

4. How many terms of the Maclaurin series for $f(x) = e^x$ are needed so that the absolute value of the error in $e^{0.2}$ is within 0.0001?

5. For what values of x is $\cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$ with the absolute value of the error less than 0.01?

6. For what values of x is $\ln(1 + x) \approx x - \frac{x^2}{2} + \frac{x^3}{3}$ with the absolute value of the error less than 0.0001?

7. Approximate $\int_0^1 \cos(x^2) dx$ using the first 3 non-zero terms of the Maclaurin series for $f(x) = \cos(x^2)$. How accurate is this approximation?