Representing Functions by Power Series- HW Problems

Find a power series representation of the following functions and determine the interval of convergence.

1. $f(x) = \frac{1}{1+x}$
2. $g(x) = \frac{1}{1-x^3}$
3. $h(x) = \frac{4}{2-x}$
4. $f(x) = \frac{8}{4+x^2}$
5. $g(x) = \frac{x^2}{x^2-4}$
6. $h(x) = \frac{x}{8+2x^2}$

Find a power series representation of the following functions through integration or differentiation and determine the interval of convergence.

7. $f(x) = \ln(3-x)$
8. $g(x) = \ln(1-x^2) = \ln(1-x) + \ln(1+x)$
9. $f(x) = \frac{1}{(1+x)^2} = \frac{d}{dx}\left(-\frac{1}{1+x}\right)$
10. $g(x) = \tan^{-1}(x^2)$ (consider $\int \frac{2x}{1+x^4}\,dx$)

Approximate the following integrals to 6 decimal places.

11. $\int_0^{0.1} \frac{1}{1+x^4}\,dx$

12. $\int_0^{0.2} \frac{x}{1+x^3}\,dx$